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Thermoelastic Problem of a Hollow Elliptical Cylinder Subjected to a Partially Distributed Heat Supply

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Abstract- This paper presents the theoretical treatment of a thermoelastic problem of a hollow elliptical cylinder due to partially distributed heat supply on the outer curved surface. Integral transform techniques have been utilized to obtain the solution having the influence of heating and cooling conditions for the problem in the form of a Mathieu series.

Key Words- Temperature distribution, thermal stress, integral transform .

Ams Subject Clasification No: 74G70, 74E05, 74A10 .

1 Introduction

This paper presents the theoretical treatment of a thermoelastic problem of a hollow elliptical cylinder due to partially distributed heat supply on the outer curved surface.

2. STATEMENT OF THE PROBLEM

We consider elliptical cylinder of inside radius a, outside radius b (where $a < b$) and thickness δ . The geometry of the cylinder indicates that an elliptic coordinate system (ξ, η, z) is the most appropriate choices of reference frame, which are related to the rectangular coordinate system (x, y, z) by the relation

$$x = c\cosh\xi\cos\eta, \quad y = c\sinh\xi\sin\eta, \quad z = z \tag{1}$$

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The curves $\eta =$ constant represent a family of confocal hyperbolas while the curves $\xi =$ constant represent a family of confocal ellipses. The length $2c$ is the distance between their common foci (refer to Figure 1). Both sets of curves intersect each other orthogonally at every point in space. The parameter ξ varies from 0 where it defines the interfocal line, to ξ_0, the coordinate η is an angular coordinate taking the range $\eta \in [0, 2\pi)$, and $z \in (0, \delta)$. It is noted that c is denoted as $2c = (a^2 - b^2)^{1/2}$ and $\xi_0 = \tanh^{-1}(b/a)$ in terms of the semi major axial-length a and the semi minor axial-length b. The heat conduction differential equation is given as

$$h^2\left(\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2}\right) + \frac{\partial^2 T}{\partial z^2} - \frac{2h_0}{\delta\lambda}(T - T_a) = 0 \tag{2}$$

where

$$h^{-2} = c^2(\cosh\xi - \cos 2\eta)/2 \tag{3}$$

Introducing the following dimensionless parameters

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$$\bar{h}^2 = h^2 a^2 ,\ \bar{h}_0 = h_0 \frac{a}{\lambda} ,\ ,\ \bar{b} = \frac{b}{a} ,\ \bar{\delta} = \frac{\delta}{a} ,\ e = \frac{c}{a} ,\ \bar{T}_a = \frac{T_a}{T_0}$$

$$\bar{T}(\xi,\eta,z) = \frac{T(\xi,\eta,z)}{T_0} ,\ \bar{\chi} = \frac{\chi}{E\alpha_r T_k a_o^2} ,\ \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{E\alpha_r T_k}\ (i,j=\xi,\eta) \tag{4}$$

Heating Process:

The equation (2) can be written in the dimensionless form as:

$$\bar{h}_1^2\left(\frac{\partial^2\bar{T}}{\partial\xi^2} + \frac{\partial^2\bar{T}}{\partial\eta^2}\right) + \frac{\partial^2\bar{T}}{\partial z^2} - \frac{4h_0 e^2}{2\bar{\delta}}(\bar{T} - \bar{T}_a) = 0 \tag{5}$$

where

$$\bar{h}_1^{-2} = (\cosh 2\xi - \cos 2\eta)/2 \tag{6}$$

and compiling various dimensionless boundary conditions are defined to determine the influence of the thermal boundary conditions on the thermal stresses as:

$$M_\xi(T,0,1,\xi_i) = R_1(\eta,z) \text{, for all } -h \le z \le h \tag{7}$$

$$M_\xi(T,0,1,\xi_0) = R_2(\eta,z) \text{, for all } -h \le z \le h \tag{8}$$

where k_1 and k_2 are the radiation constants on the two annular fin surfaces

$$M_z(T,0,1,-h) = Z_1(\xi,\eta) \text{, for all } \xi_i \le \xi \le \xi_o \tag{9}$$

$$M_z(T,0,1,h) = (Q_0/\lambda)Z_2(\xi,\eta) \text{, for all } \xi_i \le \xi \le \xi_o \tag{10}$$

Cooling Process:

On the other hand for the cooling process the temperature distribution $T'(\xi,\eta,z)$ satisfies the equation

$$\bar{h}_1^2\left(\frac{\partial^2\bar{T}'}{\partial\xi^2} + \frac{\partial^2\bar{T}'}{\partial\eta^2}\right) + \frac{\partial^2\bar{T}'}{\partial z^2} - \frac{4h_0 e^2}{2\bar{\delta}}(\bar{T}' - \bar{T}_a) = 0 \tag{11}$$

and the various dimensionless boundary conditions are

$$M_\tau(\theta',1,0,0) = M_\tau(\theta,1,0,t_0) \text{, for all } 1 \le \xi \le R,\ 0 \le \zeta \le L \tag{12}$$

$$M_\xi(\theta',1,k_1,1) = 0,\ M_\xi(\theta',1,k_2,R) = 0 \text{, for all } 0 \le \varsigma \le L,\ \tau \ge \tau_0 \tag{13}$$

$$M_\varsigma(\theta',0,1,0) = 0,\ M_\varsigma(\theta',0,1,L) = 0 \text{, for all } 1 \le \xi \le R,\ \tau \ge \tau_0 \tag{14}$$

Thus, the equations (1) to (14) constitute the mathematical formulation for heating and cooling problems under consideration.

Thermal Stresses:

The dimensionless radial and tangential stresses are given as [3]

$$\bar{\sigma}_{rr} = \bar{h}^2\frac{\partial^2\bar{\chi}}{\partial\eta^2} + \frac{e^2\bar{h}^4}{2}\sinh(2\xi)\frac{\partial\bar{\chi}}{\partial\xi} - \frac{e^2\bar{h}^4}{2}\sin(2\eta)\frac{\partial\bar{\chi}}{\partial\eta} \tag{15}$$

$$\bar{\sigma}_{\eta\eta} = \bar{h}^2\frac{\partial^2\bar{\chi}}{\partial\xi^2} - \frac{e^2\bar{h}^4}{2}\sinh(2\xi)\frac{\partial\bar{\chi}}{\partial\xi} + \frac{e^2\bar{h}^4}{2}\sin(2\eta)\frac{\partial\bar{\chi}}{\partial\eta} \tag{16}$$

$$\bar{\sigma}_{\xi\eta} = -\bar{h}^2\frac{\partial^2\bar{\chi}}{\partial\xi\,\partial\eta} + \frac{e^2\bar{h}^4}{2}\sinh(2\eta)\frac{\partial\bar{\chi}}{\partial\xi} + \frac{e^2\bar{h}^4}{2}\sinh(2\xi)\frac{\partial\bar{\chi}}{\partial\eta} \tag{17}$$

3. Solution of the problem

Determination of the heat conduction equation:

Applying integral transformation defined in [1] to the equations (5), (7), (8) over the variable z with responds to the boundary condition (9-10) one obtains

$$h^2\left(\frac{\partial^2\bar{T}}{\partial\xi^2} + \frac{\partial^2\bar{T}}{\partial\eta^2}\right) - \frac{2h_0}{\delta\lambda}(\bar{T} - \bar{T}_a) = 0 \tag{18}$$

$$M_\xi(\bar{T},0,1,\xi_i) = R_1(\eta,z) \tag{19}$$

$$M_\xi(\bar{T},0,1,\xi_0) = R_2(\eta,z) \tag{20}$$

Where $\bar{T}$ denotes the transformed function of T, a_n are the eigenvalues of the transcendental equations

$$[\alpha_1 a\cos(ah) + \beta_1\sin(ah)][\beta_2\cos(ah) + \alpha_2 a\sin(ah)]$$

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$$= [\alpha_2 a \cos(ah) - \beta_2 \sin(ah)][\beta_1 \cos(ah) - \alpha_1 a \sin(ah)] \tag{21}$$

$\alpha_1 = \bar{k}_1$, $\beta_1 = 1$, $\alpha_2 = \bar{k}_2$, $\beta_2 = 1$ responds to the boundaries conditions of type (12.2.8) as

$$M_z(T, \beta_1, \alpha_1, h) = 0, \; M_z(T, \beta_2, \alpha_2, -h) = 0 \tag{22}$$

and

$$\bar{f}(n,t) = \int_{-h}^{h} f(z,t) P_n(z)\, dz \tag{23}$$

Here the kernel is given by the orthogonal functions in the interval $-h \le z \le h$ as

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z) \tag{24}$$

where

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_2 - \alpha_1) a_n \sin(a_n h)$$

4. CONCLUSION

In this paper presents the theoretical treatment of a thermoelastic problem of a hollow elliptical cylinder due to partially distributed heat supply on the outer curved surface.

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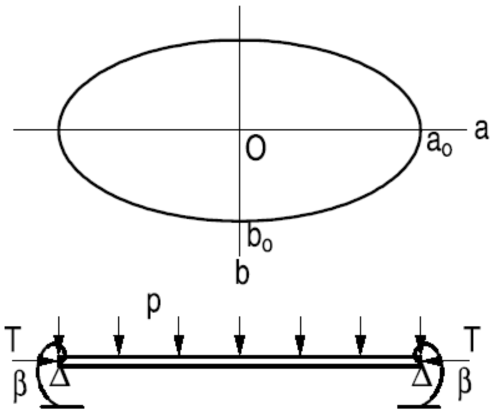

Fig. 1 Geometry of the problem

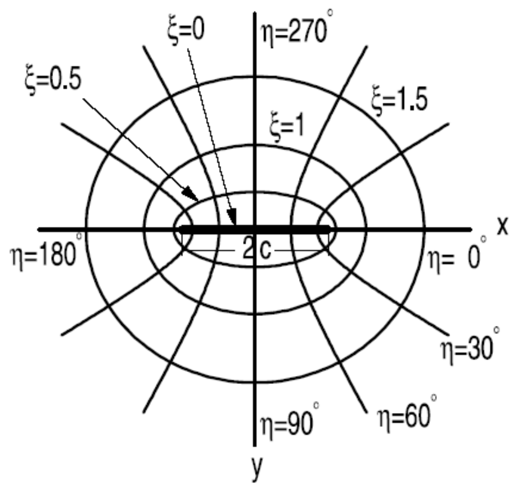

Fig. 2 The elliptical coordinate system